

MAT 534: HOMEWORK 4
DUE W SEP 23

Problems marked by asterisk (*) are optional.

1. Let $K, L \leq G$ be such that L normalizes K , i.e., $lKl^{-1} = K$ for all $l \in L$. Prove that $KL \leq G$ and $KL \cong K \rtimes L/K \cap L$.
2. Describe all Sylow 2-subgroups and 3-subgroups of D_{12} .
3. Prove that if $|G| = 105$, then G has a normal Sylow 5-subgroup and a normal Sylow 7-subgroup. Prove that if G has a normal Sylow 3-subgroup, then G is abelian.
4. Let G be a group of order p^2q , where p, q are primes, $p < q$. Assume that p does not divide $q - 1$. Prove that then G is abelian.
- 5*. Classify all groups of order 75.
- 6*. Classify all groups of order 20.